

Pole Analysis of Unitarized One Loop χ PT Amplitudes – A Triple Channel Study

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Abstract

In a previous paper [1], we proposed a method to distinguish poles of different dynamical origin, in a unitarized amplitude of $\pi\pi$, $K\bar{K}$ system. That is based on the observation that ‘A Breit-Wigner resonance should exhibit two poles on different Riemann sheets which meet each other on the real axis when $N_c = \infty$ ’. In this paper, we extend our previous work [1] to the $\pi\pi$ - $K\bar{K}$ - $\eta\eta$ three channel system. We reconfirm most of the predictions of Ref. [1]. Especially the $f_0(980)$ is of $K\bar{K}$ molecule nature. Other poles, including the σ , are of Breit-Wigner type.

Key words: Meson – Meson scattering, Unitarity, Hadron resonance

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1 Introduction

Hadron physics in the intermediate energy region (i.e., $\sim 0.5 - 2\text{GeV}$) is well known to be a difficult field of study, since neither pQCD nor chiral perturbation theory (ChPT) is

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applicable there. Especially the study on the property of light scalar mesons has a long history of controversy. Dispersion technique is found to be powerful, in firmly establishing the existence of light scalar mesons such as σ , κ and in determining rather precisely their pole locations [2]-[7]. However, the property of the light scalar mesons ($f_0(980), a_0(980), f_0(600), K_0^*(700)$) remains somewhat mysterious.¹

One commonly used method in the literature to attack this problem is the so called “Inverse Amplitude Method” or “chiral unitarization approach”. The $\pi\pi - K\bar{K}$ and $\pi\eta - K\bar{K}$ couple channel system is extensively studied [9]-[12], giving the sheet II trajectory of light mesons [11]-[15] below 1GeV. In Ref. [1] it is pointed out that a couple channel Breit–Wigner resonance should exhibit two poles on different Riemann sheets and reach the same position on the real axis when $N_c = \infty$. In this point of view ‘searching for accompanying shadow pole’ is stressed which is overlooked by most previous studies. An advantage of this method is that the criteria is unaltered when changing parameters and hence avoids parameter dependence. The worry of being model (Padé approximation) dependent is also, at least partly, avoided, since one can examine the pole structure of well known particles (such as ρ and K^* resonances) to justify the validity of using Padé approximation in studying analyticity property of the pole. In this paper we follow the idea of Ref. [1] and extend it to the situation with $\pi\pi, K\bar{K}, \eta\eta$ three channel system (together with $\pi\eta, K\bar{K}$ system when studying $a_0(980)$). We find that, with $\eta\eta$ channel included, qualitative results obtained in Ref. [1] remain valid. Especially, it is found that the $f_0(980)$ pole remains to be a single trajectory. Nevertheless, we point out that a careless mistake occurs in Ref. [1]: the σ, κ were mistakenly claimed to have only single trajectory in two channel case. According to a careful re-analysis, they both contain a shadow pole on sheet III – hence the σ, κ mesons are also of couple channel Breit – Wigner resonance origin.

This paper is organized as follows. In section 2 we give a brief introduction to partial wave projection, analytical continuation, and Padé unitarization. In section 3 we fit the experimental phase shift and inelasticity to fix the low energy constants (LECs) and extract pole positions. In section 4 we analyze the trajectory of the poles. The last section is for

¹This problem is discussed, for example, in Ref. [8].

conclusion.

2 Unitarization

2.1 Partial wave expansion

The ChPT scattering amplitudes have been given in [11, 16] and the iso-spin decomposed formulas are given in [1]: The projection in definite angular momentum is given by

$$T^{(I,J)} = \frac{1}{32N\pi} \int_{-1}^1 d(\cos\theta) T^I(s, \cos\theta) P_J(\cos\theta) \quad (1)$$

where I is isospin, J denotes the total angular momentum. Here $N = 2$ for $\pi\pi, \eta\eta \rightarrow \pi\pi, \eta\eta$ amplitudes, $N = \sqrt{2}$ for $\pi\pi, \eta\eta \rightarrow K\bar{K}$ amplitudes, and $N=1$ for other cases. From now on we omit the I,J index for simplicity.

2.2 Padé approximation

Matrix Padé approximation was derived to obtain unitarized amplitudes from the $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ ChPT scattering amplitudes, which is equivalent to:

$$T = T^{(2)} \cdot [T^{(2)} - T^{(4)}]^{-1} \cdot T^{(2)} \quad (2)$$

For a three channel case, the unitarized amplitudes satisfy such unitarity conditions,

$$\begin{aligned} \text{Im}_R T_{11} &= T_{11}\rho_1 T_{11}^* \theta(s - 4m_\pi^2) + T_{12}\rho_2 T_{12}^* \theta(s - 4m_K^2) + T_{13}\rho_3 T_{13}^* \theta(s - 4m_\eta^2) , \\ \text{Im}_R T_{12} &= T_{11}\rho_1 T_{12}^* \theta(s - 4m_\pi^2) + T_{12}\rho_2 T_{22}^* \theta(s - 4m_K^2) + T_{13}\rho_3 T_{23}^* \theta(s - 4m_\eta^2) , \\ \text{Im}_R T_{13} &= T_{11}\rho_1 T_{13}^* \theta(s - 4m_\pi^2) + T_{12}\rho_2 T_{23}^* \theta(s - 4m_K^2) + T_{13}\rho_3 T_{33}^* \theta(s - 4m_\eta^2) , \\ \text{Im}_R T_{22} &= T_{12}\rho_1 T_{12}^* \theta(s - 4m_\pi^2) + T_{22}\rho_2 T_{22}^* \theta(s - 4m_K^2) + T_{23}\rho_3 T_{23}^* \theta(s - 4m_\eta^2) , \\ \text{Im}_R T_{23} &= T_{12}\rho_1 T_{13}^* \theta(s - 4m_\pi^2) + T_{22}\rho_2 T_{23}^* \theta(s - 4m_K^2) + T_{23}\rho_3 T_{33}^* \theta(s - 4m_\eta^2) , \\ \text{Im}_R T_{33} &= T_{13}\rho_1 T_{13}^* \theta(s - 4m_\pi^2) + T_{23}\rho_2 T_{23}^* \theta(s - 4m_K^2) + T_{33}\rho_3 T_{33}^* \theta(s - 4m_\eta^2) . \end{aligned} \quad (3)$$

Here the subscript 1 represents $\pi\pi$, 2 for $K\bar{K}$, and 3 represents $\eta\eta$. The fourth equation only holds true above $4m_K^2 - 4m_\pi^2$ [17] and the sixth equation only holds true above $4m_\eta^2 - 4m_\pi^2$.

The same as in couple channel, it violates unitarity as the left hand cut $(-\infty, 4m_\eta^2 - 4m_\pi^2]$ appears not only in T_{33} , but also in the other five T matrix elements.

Partial wave S matrix elements are given by

$$\begin{aligned}
S_{11} &= 1 + 2i\rho_1(s)T_{11}(s) , \\
S_{12} &= 2i\sqrt{\rho_1(s)\rho_2(s)}T_{12}(s) \\
S_{13} &= 2i\sqrt{\rho_1(s)\rho_3(s)}T_{13}(s) \\
S_{22} &= 1 + 2i\rho_2(s)T_{22}(s) , \\
S_{23} &= 2i\sqrt{\rho_2(s)\rho_3(s)}T_{23}(s) \\
S_{33} &= 1 + 2i\rho_3(s)T_{33}(s) .
\end{aligned} \tag{4}$$

The 3×3 S-matrix parametrization is given in the paper [18, 12]:

$$\Phi(x) = \begin{pmatrix} \eta_{11} \exp^{2i\delta_{11}} & \eta_{12} \exp^{i\delta_{12}} & \eta_{13} \exp^{i\delta_{13}} \\ \eta_{12} \exp^{i\delta_{12}} & \eta_{22} \exp^{2i\delta_{22}} & \eta_{23} \exp^{i\delta_{23}} \\ \eta_{13} \exp^{i\delta_{13}} & \eta_{23} \exp^{2i\delta_{23}} & \eta_{33} \exp^{2i\delta_{33}} \end{pmatrix} . \tag{5}$$

2.3 Analytical Continuation

To extend the scattering amplitudes in the complex plane, we need analytic continuation. The multi-channel continuation has been given by the paper according to the cuts caused by phase space factor ρ [19]. Here we use the following order of analytical continuation for simplicity:

The first continuation : $T^I(s - i\epsilon) = T^{II}(s + i\epsilon)$,

The second continuation : $T^I(s - i\epsilon) = T^{III}(s + i\epsilon)$, $T^{II}(s - i\epsilon) = T^V(s + i\epsilon)$,

The third continuation : $T^I(s - i\epsilon) = T^{IV}(s + i\epsilon)$, $T^{II}(s - i\epsilon) = T^{VIII}(s + i\epsilon)$,

$T^{III}(s - i\epsilon) = T^{VI}(s + i\epsilon)$, $T^V(s - i\epsilon) = T^{VII}(s + i\epsilon)$.

They generate the Riemann sheets:

| | I | II | III | IV | V | VI | VII | VIII |
|----------|---|----|-----|----|---|----|-----|------|
| ρ_1 | + | - | - | - | + | + | - | + |
| ρ_2 | + | + | - | - | - | + | + | - |
| ρ_3 | + | + | + | - | + | - | - | - |

Table 1: The definition of Riemann sheets.

3 Fixing LECs and pole trajectory

3.1 Fit of all channels

We fit data in $IJ = 00$ $\pi\pi$ - $K\bar{K}$ - $\eta\eta$ triple channel, $IJ = 20$ $\pi\pi$ single channel, $IJ = \frac{3}{2}0$ πK single channel, $IJ = 11$ $\pi\pi$ - $K\bar{K}$ couple channel, $IJ = 10$ $\pi\eta$ - $K\bar{K}$ couple channel, and $IJ = \frac{1}{2}0, \frac{1}{2}1$ πK - ηK couple channel. The data we used are as follows: for $IJ = 00$, the $\pi\pi \rightarrow \pi\pi$ phase is from [20, 21], $\pi\pi \rightarrow K\bar{K}$ phase from [22, 23]; $IJ = 11$ $\pi\pi \rightarrow \pi\pi$ phase from [24]; $IJ = 20$ $\pi\pi$ phase from [20]; $IJ = \frac{3}{2}0$ πK phase from [25]; and $IJ = \frac{1}{2}0, \frac{1}{2}1$ $\pi K \rightarrow \pi K$ phase from [26, 27]. For $IJ = 10$, we use the $\pi\eta$ effective mass distribution data from the $pp \rightarrow p(\eta\pi^+\pi^-)p$ reaction studied by WA76 Collaboration [28], with the background given by [29]. Here we use the same spectrum function as that of [11],

$$\frac{d\sigma}{dE_{cm}} = cp_{\pi\eta}|T_{\pi\eta \rightarrow K\bar{K}}^{IJ=10}|^2 + b.g. , \quad (6)$$

where c is a normalization factor. In additional, the data we fit are among the energy region $4m_\pi^2 - 1.2GeV$, which is much higher than the validity domain of ChPT. Parameters other than LECs are given by: $f_\pi = 0.0924GeV$, $m_\pi = 0.1373GeV$, $m_K = 0.4957GeV$, $m_\eta = 0.5475GeV$, and the renormalization scale of ChPT amplitudes is taken to be $\mu = 0.7755GeV$. The numerical results are shown in Table 2 and Fig. 1. We also list the value of ChPT and results obtained in Ref. [1] in Table 2 for comparison. We find that the LECs obtained from $IJ = 00$ $\pi\pi$ - $K\bar{K}$ - $\eta\eta$ triple channel fit can also reproduce the $\pi\pi$ - $K\bar{K}$ couple channel data very well.

With these LECs we get poles and their trajectories generated by running N_c . The N_c trajectory of poles are plotted in Fig. 2 c)–h). The pole locations when $N_c = 3$ are presented

| | Fit | Ref. [1] | ChPT (10^{-3}) |
|--------------------|------------------|------------------|--------------------|
| L_{1r} | 1.18 ± 0.01 | 1.25 ± 0.02 | 0.4 ± 0.3 |
| L_{2r} | 1.67 ± 0.01 | 1.63 ± 0.03 | 1.4 ± 0.3 |
| L_{3r} | -3.91 ± 0.01 | -4.07 ± 0.02 | -3.5 ± 1.1 |
| L_{4r} | -0.22 ± 0.01 | 0.30 ± 0.02 | -0.3 ± 0.5 |
| L_{5r} | -0.99 ± 0.08 | -1.44 ± 0.42 | 1.4 ± 0.5 |
| $2L_{6r} + L_{8r}$ | -0.50 ± 0.03 | 0.32 ± 0.13 | 0.5 ± 0.7 |
| $2L_{7r} + L_{8r}$ | 2.26 ± 0.15 | 1.38 ± 0.38 | 0.5 ± 0.7 |

Table 2: LECs obtained from fit.

in Table 3. For comparison, we also use these LECs to plot trajectories of $f_0(980)$, σ in the old scheme of Ref. [1]. The plots are in Fig. 2a) – b). The pole locations when $N_c=3$ are presented in Table 3, too. In the present case, the eight-sheets structure is rather

| Resonance | II | III | IV | VII |
|---------------|------------------|------------------|------------------|------------------|
| 2×2 | | | | - |
| $f_0(980)$ | $0.989 - i0.005$ | | | - |
| $a_0(980)$ | | $0.707 - i0.174$ | $0.930 - i0.442$ | - |
| σ | $0.441 - i0.238$ | $0.398 + i0.130$ | | - |
| $\kappa(700)$ | $0.746 - i0.192$ | $0.602 + i0.263$ | | - |
| $K^*(892)$ | $0.871 - i0.020$ | $0.903 - i0.017$ | | - |
| $\rho(770)$ | $0.760 - i0.070$ | $0.797 - i0.058$ | | - |
| 3×3 | | | | |
| $f_0(980)$ | $0.990 - i0.004$ | | | |
| σ | $0.462 - i0.230$ | $0.400 - i0.194$ | $0.397 - i0.128$ | $0.453 - i0.235$ |

Table 3: Resonance pole positions on \sqrt{s} plane, in units of GeV.

complicated for presenting results. Confined to simplified two channel situation, it is found that the trajectories of ρ , K^* , $a_0(980)$, $f_0(980)$ are qualitatively the same as our previous work [1]. It is worth pointing out that for κ and σ , a sheet III pole overlooked in Ref. [1] was found. We find that these two poles satisfy the twin pole structure: poles in sheet II,III meet each other in the real axis when $N_c = \infty$ (see Fig. 2b) and e). On the other side,

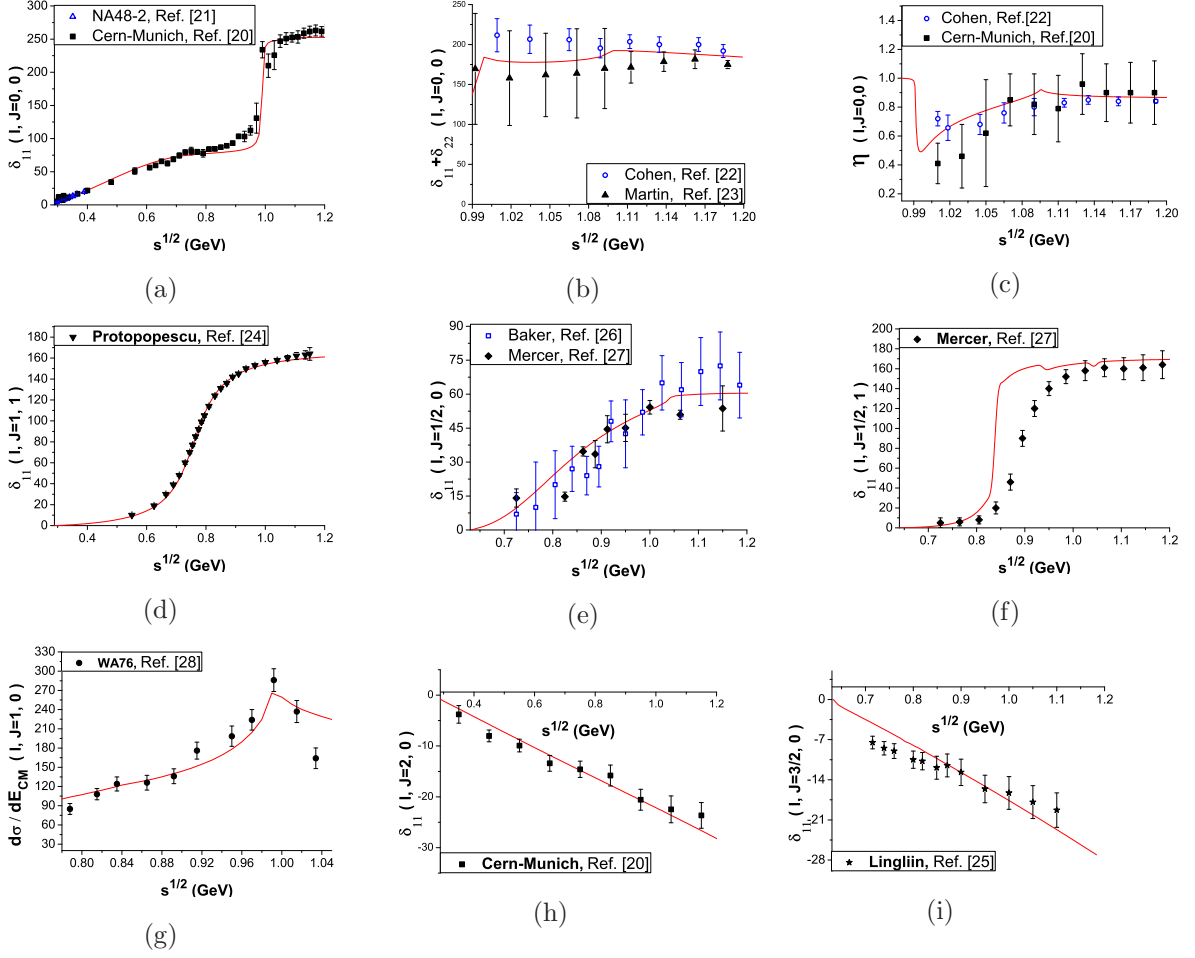
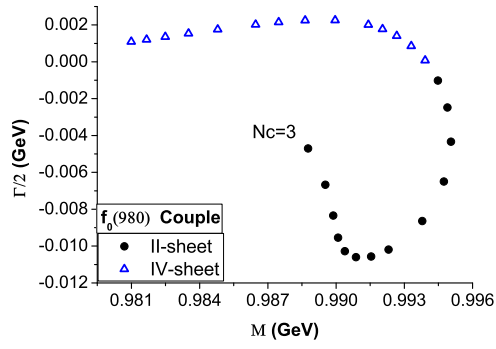
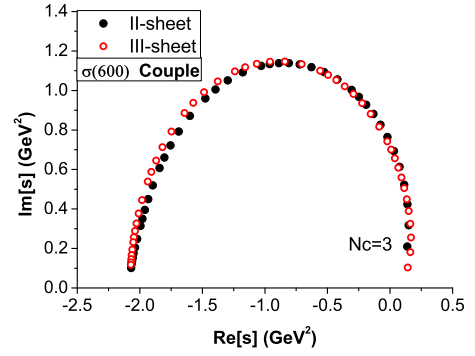


Figure 1: Fit phase shift, inelasticity and cross section.

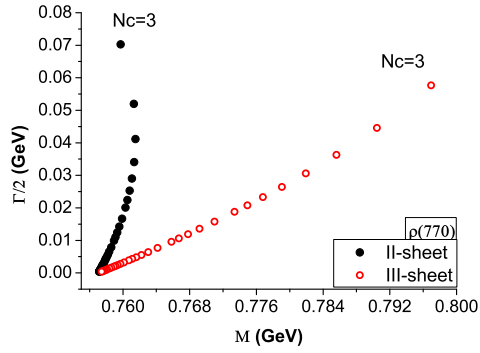
for $f_0(980)$, even discussing in the triple channel case, its trajectory is like what is found in Ref. [1]: the sheet II pole moves into upper half plane of sheet V from the lower half of sheet II in the \sqrt{s} -plane, winding around the branch point at the $K\bar{K}$ threshold (see Fig. 2a), g)). Eventually it will fall down to the positive real axis, between $\pi\pi$ and $K\bar{K}$ threshold, with *no* shadow poles accompanied. The trajectory of σ in the triple channel study exhibits a clear quadruplet pole structure: there are four adjoint poles in sheet II, III, IV, VII, and they will meet each other in the negative real axis of the s complex plane when $N_c = \infty$ (see Fig. 2h)).



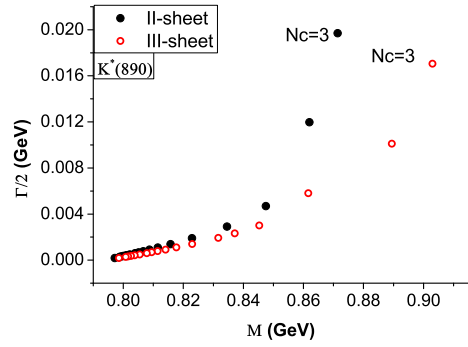
(a)



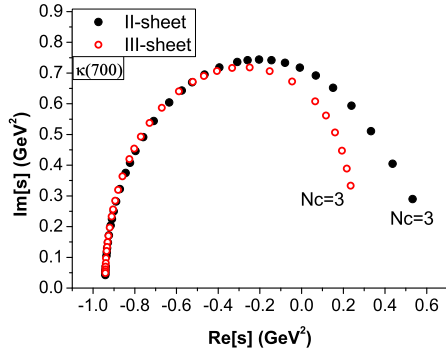
(b)



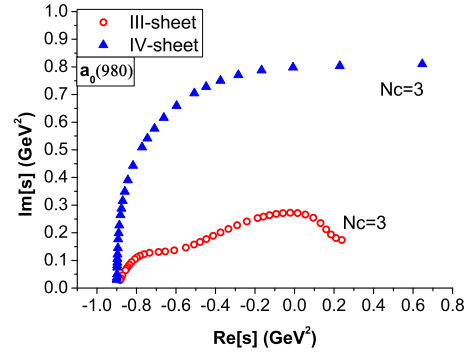
(c)



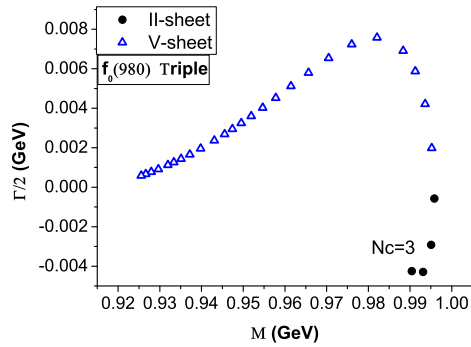
(d)



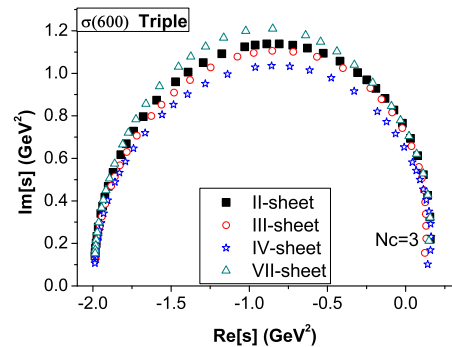
(e)



(f)



(g)



(h)

Figure 2: N_C trajectories.

4 Conclusion

In this paper, we have made a rather sophisticated analysis on the analyticity structure of Padé amplitudes, with $IJ=00$ $\pi\pi-K\bar{K}-\eta\eta$ channels included. Our ‘Breit-Wigner criteria’ previously proposed in Ref. [1] is extended to multi-channel case: A triple channel Breit-Wigner resonance should appear as quadruplet poles and meet each other on the real axis when $N_C = \infty$. Our analysis confirm that ρ , K^* are standard Breit-Wigner particles due to their stable twin pole structure. For $a_0(980)$ one needs more precise data to get a better pole location when $N_c = 3$ and our analysis supports it to be a Breit-Wigner resonance.

For $f_0(980)$, we find that the qualitative picture and physical conclusion one can draw from this analysis is very similar to our previous results given in Ref. [1]. It reveals that $f_0(980)$ plays a very special role in the family of low lying scalar mesons: it is most likely of $K\bar{K}$ nature. The meaning of this paper is that we demonstrate such a distinguishing property of $f_0(980)$ is stable against the number of thresholds it couples.

We also corrected the results on σ and κ made in Ref. [1]. Here we find that both σ and κ maintain a shadow pole structure, hence revealing their Breit-Wigner origin, even though they behave very oddly in reality and in the large N_c world. Nevertheless the similarities between two trajectories in sheet II and sheet III even when N_C is not large suggests that they are dominantly single channel particles.

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